

The Effect of Kit-Build Concept Mapping for Organizing Information on Conceptual and Procedural Knowledge in Mathematics: A Comparative Study

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Abstract—Developing both conceptual and procedural knowledge is important for learning mathematics. It is insufficient to rely only on memorizing learned facts or principles for solving a complex problem. Additionally, conceptual understanding may affect the acquisition of procedural knowledge. Thus, developing conceptual knowledge needs to be focused on early in the learning process. Writing a structured summary is one of many methods that may help learners organize their understanding and develop conceptual knowledge. However, the currently popular method of providing an after-class activity of writing a structured summary of learning materials is insufficient, which often leads to retained misconceptions. This study investigates the potential effectiveness of activity for organizing information that utilizes a Kit-Build Concept Mapping (KBCM) tool as an alternative for developing both conceptual and procedural knowledge. An experiment was conducted to compare KBCM with summary writing, which involved 105 students from an Indonesian public university. The result suggests the potential of KBCM to improve conceptual knowledge better than summary writing. In addition, both treatments were found to be potentially effective for improving procedural knowledge. Some characteristics of KBCM that might have played a role in developing both types of knowledge are discussed.

Keywords—conceptual knowledge, procedural knowledge, kit-build concept mapping, summary writing, teaching mathematics

I. INTRODUCTION

In mathematics educational contexts, learners are usually expected to solve problems by recognizing the nature of the problem, retrieving relevant principles, and then applying relevant procedures, i.e., they are expected to acquire understanding about both conceptual knowledge and procedural knowledge. Understanding conceptual knowledge implies understanding “what” and “why” certain principles in mathematics work [1], while understanding procedural knowledge implies the ability to memorize, retrieve, and apply the knowledge about steps or “how to solve” a problem. In the field of mathematics education, some works of literature suggested the importance of developing both types of knowledge for effective problem solving [2–4].

However, while procedural knowledge needs to be fostered, a primary focus needs to be given to developing conceptual knowledge as early as a new concept is introduced to learners due to its influential role in shaping procedural knowledge [3, 5, 6].

However, despite its importance, developing conceptual knowledge remains a difficult challenge for many instructors. In the context of mathematics education, learners sometimes perform mathematical procedures involving certain mathematical objects (operators, symbols, terms, etc.) but do not know what the underlying principles are, including the reasons behind the procedures and their relationships with other mathematical concepts. In addition, gaining a “big picture” about mathematical concepts (i.e., relationships between key concepts) had also been suggested in prior works of literature [7–9]. A study even reports that a significant number of lectures for teaching mathematics still put an emphasis on providing procedural knowledge instruction (e.g., focusing on memorization) instead of conceptual knowledge [10].

As for teaching conceptual knowledge, several methods have been proposed, such as providing interactive engagement, teach-to-learn assignments, visualizations, examples and non-examples, and guided discovery [11–13]. However, traditional lecturing, as well as the mentioned alternative methods, have some limitations, especially in a class with diverse learners, e.g., learners who retain learned misconceptions, low-achieving learners (low-achievers), etc. As an example, some low-achievers may have difficulty catching the meaning and relationships between abstract concepts just by listening to a lecture. It may be difficult for low-achievers to complete teach-to-learn assignments due to a relatively higher cognitive load.

Another typical way to help learners develop conceptual knowledge is by instructing them to write about the learning materials, e.g., essay assignments, summaries, etc., usually as after-class activities to help learners organize their understanding [14]. Writing a summary about a learning material has long been viewed as beneficial for learners due to its potential to improve comprehension [15–17]. In mathematics teaching, giving writing assignments is believed to be beneficial for conceptual knowledge development because it requires learners to make efforts to achieve a thorough understanding of various concepts in order to effectively express and explain them to others [9, 18]. Such an activity also requires learners to catch the structure of the domain knowledge and express it using their own words.

Another alternative that could help the learners gain conceptual knowledge is concept mapping, in which learners are required to extract and express the structure of the

knowledge in the form of propositions represented as links and nodes [19, 20]. Through writing a summary or creating a concept map, learners could actively organize information by extracting propositions (e.g., definitions & examples) from a knowledge domain and be aware of the relationships between concepts.

Nevertheless, both summary writing and making concept maps, especially traditional from-a-scratch concept mapping, may not also be befitting for those who are low achievers. Both of them require the learners to extract the propositions from the learning materials, restate the propositions in their own words, and recognize underlying relationships between the concepts, which demand a relatively higher cognitive load. In addition, learners may also be prone to misconceptions during summary writing or concept mapping, which may further hinder their learning if left unaddressed. In the context of the teaching of linear algebra, misconceptions due to learners' inability to correctly recognize relationships between important concepts have been reported in prior studies [21, 22]. Therefore, there is a need to provide an effective alternative method as a complement to the existing way of teaching conceptual knowledge, which requires learners to develop and retain a correct understanding of conceptual knowledge from the start of the learning process.

Another alternative activity for organizing information that could address the limitations of summary writing and traditional from-scratch concept mapping is Kit-Build Concept Mapping (KBCM). In this approach, firstly, the instructors prepare a concept map representing the domain knowledge. Secondly, the teacher-created map is disassembled into components (nodes & links) [23]. Such components are then given to the learners, who are instructed to try to reconstruct the map while reviewing the materials without seeing the teacher-created map. They receive immediate feedback while attempting to connect the nodes and links; thus, they have the opportunity to reflect on their understanding. Such a way is hypothesized to be beneficial for developing conceptual knowledge, i.e., they are aware of the relationships between the concepts within the domain. KBCM has been applied in various educational contexts with positive effects on learning [24–28].

Nevertheless, the question of what knowledge type KBCM could improve and to what degree it is effective compared to typical summary writing as an after-class activity has not been addressed in prior studies about KBCM. A related prior study [28] provided the evidence that KBCM has the potential to improve overall learning achievement in mathematics. However, the mentioned study has not yet investigated its effect on learners' conceptual and procedural knowledge. To address this research gap, this study aims to investigate the effectiveness of KBCM as an alternative way to improve learners' conceptual and procedural knowledge, particularly in the context of learning mathematics. To achieve this purpose, this study compares the effectiveness of providing KBCM and the typically implemented summary writing as an after-class information-organizing activity for improving learners' conceptual knowledge. By comparing the mentioned interventions (KBCM and summary writing), the following research question is addressed in this study: To what extent did conceptual and procedural knowledge

improve?

II. RELATED WORKS

A. Conceptual and Procedural Knowledge

Conceptual knowledge is a web of ideas that form meaning, e.g., relationships between newly learned concepts and previously learned concepts, propositions describing related objects that convey a principle or theory, etc. [29, 30]. In the domain of linear algebra, for example, this type of knowledge includes the definition that states a vector could be any mathematical object as long as it satisfies a certain set of axioms, i.e., recognizing the existence of a relationship between the concept of algebraic vectors and the mentioned axioms.

On the other hand, procedural knowledge is the knowledge about a series of actions or steps needed to be performed to reach a goal, e.g., steps to apply relevant mathematical formulas in a specific context [29]. An example of this type of knowledge in the domain of linear algebra includes the knowledge of steps and the principles for the addition of two polynomials, e.g., $x^2 + x$ and $x^3 + 2x$.

Conceptual knowledge plays an important role in shaping procedural knowledge [3]. Thus, having a conceptual misunderstanding of a concept is detrimental to the development of problem-solving abilities. As an illustration in the context of learning mathematics, a learner might have acquired an understanding of the concept of vectors as physical vectors describing physical forces represented in a two-dimensional (e.g., $\mathbf{x} = [x_1, x_2]$) or three-dimensional plane (e.g., $\mathbf{x} = [x_1, x_2, x_3]$) due to being taught so in their high school physics class. When encountering algebraic vectors in a linear algebra class during their university years, they might have difficulties differentiating physical vectors and vectors as a mathematical object that satisfies certain axioms (e.g., any element within a set of polynomials P , matrices $M_{m \times n}$, or functions F that only include elements with identical algebraic properties). Such a misconception could be retained and hinder deeper learning in linear algebra [22, 31]. Therefore, in the context of mathematics education, it is important to develop the correct conceptual knowledge early.

B. Activities of Organizing Information as Instructional Strategies

Many instructional strategies have been proposed to help learners learn mathematics. The state-of-the-art works of literature in the field of mathematics education show that game-based learning and activities of organizing information (e.g., writing and concept mapping) are popular teaching interventions with various benefits and challenges [32–34]. Some of the mentioned interventions provide various forms of scaffolding to lower cognitive load and help learners effectively gain conceptual understanding to solve mathematical problems.

Organizing information is one of the ways to develop conceptual knowledge, which may affect procedural knowledge in the end. Such kinds of activities require learners to engage deeply with the learning materials in such a way that they try to memorize specific terms and symbols used to define a concept, catch the relationships between the concepts, make sense of propositions, and express their

understanding using various mediums, e.g., written text, diagrammatic representation of concepts, etc.

The activity of organizing information may facilitate meaningful learning. According to the theory of meaningful learning, learners learn effectively when they are able to integrate new knowledge into their existing understanding [35, 36]. In the context of mathematics education, summary writing and concept mapping are two forms of information organizing activities that have been used as promising instructional strategies [33, 37].

C. Summary Writing

Writing a summary of learning materials is one of the typical ways to organize information for achieving conceptual understanding. Writing to organize information has been suggested as beneficial for learning mathematics because it requires learners to try to explain various abstract concepts in their own words [9, 18].

In mathematics education, writing about learning contents is one of the pedagogical tools that have the potential to benefit learners [38–40]. Writing a summary in this context may enable learners to reflectively think and express what they have understood well and what they do not understand, which could be used as inputs for diagnosing misconceptions [41]. Learning mathematics is generally an incremental experience in which learners build their knowledge over their existing knowledge structure. Summary writing may deepen learners' understanding by helping learners realize the connection between their prior understanding, new information they received, and their life experiences [42]. This kind of writing serves as a means for actively assimilating and constructing mathematical knowledge [43].

Writing a summary about a topic in mathematics is a form of mathematical writing, which includes choosing appropriate terms to describe a mathematical concept, mixing words and symbols in a meaningful way, and synthesizing materials [44]. There are various tasks that could be included in a summary writing assignment for learning mathematics, such as (a) inquiring about existing understanding of mathematical concepts [43], (b) exploring mathematical concepts (exploratory writing) [33], and (c) documenting the learning process (journal writing) [45, 46]. Various forms of scaffolding can also be provided to support learners in performing the mentioned summary writing tasks, such as providing a structured summary template consisting of keywords as a hint for writing propositions [28]. However, despite this kind of scaffolding, summary writing is often a difficult and cognitively demanding task for learners who cannot effectively organize information because it involves extracting the knowledge structures from the learning materials by themselves.

D. Kit-Build Concept Mapping (KBCM)

Another alternative is concept mapping, which utilizes a diagram to represent propositions as a web of interconnected nodes and links [20]. Prior studies had suggested that concept maps could be used and are beneficial in assessing and teaching conceptual knowledge [47–51].

The traditional scratch concept mapping, which involves making a concept map about the learning material from scratch, has been extensively used in the context of

mathematics education [37, 52]. Various studies in mathematics education reported that scratch concept mapping has the potential to increase overall learning performance [50], conceptual knowledge [32, 53], and assessment of self-efficacy [50]. Visualized connections between nodes that represent important concepts are suggested to be a scaffold that helps increase awareness of the relationships between relevant concepts [32, 54].

Through making a concept map, learners are enabled to find the relationships between several concepts by making connections and to reflect on their previously acquired understanding of such connections [55]. However, creating a concept map from scratch using learners' own words is difficult due to a high cognitive load, which may hinder low achievers from deeper learning [20].

Despite being a scaffold itself by providing a visual representation of propositions that has the potential to improve conceptual understanding [56], there are several forms of scaffolding that could be integrated into concept mapping activities, such as providing focus questions to guide the making of propositions [57] and providing components. There are various ways of providing components as a scaffold in concept mapping, such as: (a) providing a partially completed map for learners to extend, (b) a map kit or disconnected components (fully or partially disconnected nodes & links) for learners to assemble, and (c) concept map structures without text for learners to select and fill in (SAFI) [36]. However, despite seeming restrictive and possibly limiting opportunities to express unique ideas, providing components as constraints in concept mapping could be effective for ensuring alignment of learners' understanding with expert understanding if appropriate and timely feedback is provided based on learners' misconceptions [36]. In addition, giving freedom to freely create concept maps from scratch could risk retained misconceptions due to the possibility of making incorrect propositions or poorly organized concept map structures [58]. Low achievers often experience difficulties in exercising higher-order thinking and synthesizing information, which are required for making an excellent concept map [59]. Therefore, similar to summary writing, scratch concept mapping risks an ineffective attempt to organize information due to requiring learners to exert a cognitively demanding effort to extract the knowledge structures from the learning materials by themselves.

KBCM is a type of concept mapping that could potentially help both low- and high-achievers by making the learners reconstruct a teacher-created concept map from given components, thus reducing their cognitive load when making the map. In KBCM, learners are given components (nodes & links) that are prepared in advance by disassembling a teacher-created concept map [23]. During the attempt, the learners connect the given nodes and links without seeing the teacher-created map while receiving immediate feedback on whether they connected the correct propositions [60]. Through KBCM, learners could try to catch the relationships between several concepts and reflect on their understanding by being scaffolded with the provision of given components, i.e., they are not required to extract the proposition from the learning materials by themselves and could focus on finding the correct connections between concepts.

KBCM has been used as a pedagogical method for improving understanding of mathematics in some prior studies [22, 25, 28]. The use of KBCM for teaching mathematics is aimed at increasing awareness of the connections between various important concepts (crosslinks), which is important for effective problem solving. As an example, in the context of teaching linear algebra, the learners are expected to be aware of the relationship between linear equations and inverse matrices to be successful in making a correct judgment regarding the solution of a system of linear equations. By providing KBCM activity, as shown in a prior study [28], reassembling the components representing the mentioned concepts has the potential to improve overall understanding of the learning material by reducing cognitive loads to identify and connect important concepts.

Some prior studies have shown that KBCM is effective for improving overall understanding of mathematics compared to traditional summary writing or reciprocal scratch concept mapping [25, 28]. However, none of the mentioned prior studies investigate the effectiveness of KBCM for improving conceptual or procedural knowledge in the context of mathematics education. In a quest to find effective methods for learning conceptual and procedural knowledge, this study aims to investigate the effectiveness of KBCM as an alternative to the traditional summary writing for improving conceptual and procedural knowledge in the mathematics subject.

In the present study, in order to answer the research question (to what extent did conceptual and procedural knowledge improve?), the following variables were captured from an experiment involving a KBCM-treated group and a summary writing-treated group:

- 1) conceptual knowledge understanding;
- 2) procedural knowledge understanding.

Variables (a) and (b) were measured by the pre-test and post-test scores as well as the standardized learning gain for the respective types of knowledge. The differences between the pre-test and the post-test scores within groups and across groups, as well as the difference in the standardized learning gain, were used as the evidence of the learning effect. Additionally, the correlations between the map scores, the pre-test, and post-test scores (both conceptual & procedural knowledge) of the KBCM group were calculated as the evidence of the relationship between the quality of learners' concept mapping attempts and the learning effect. A detailed description of how the experiment was conducted is provided in the following Methods section.

III. METHODS

This case study aims to investigate KBCM's effect on learners' conceptual and procedural understanding as a follow-up study of a related study that compared the effectiveness of KBCM and summary writing for improving overall learning [28]. This study implemented a quantitative quasi-experimental approach to evaluate the proposed KBCM approach. The learning context, the participants, the steps undertaken during the experiment and the analysis are described in the following sections.

A. Participants, Learning Context, and Evaluation

Students enrolled in two linear algebra classes (Class A: 54

students; Class B: 51 students) from a public Indonesian university were involved in the experiment. All students were from the computer science department. Both classes received the same learning materials about the topic of inverse matrices and elementary row operations. Two instructors who taught the classes (one instructor per class) closely worked together to ensure that the two classes received identical teaching styles and learning content.

The learning topic was inverse matrices, which is one of the key topics in linear algebra. The goals of the learning activity in this topic include: (a) the learners can describe the characteristics of inverse matrices; (b) the learners are able to infer the nature of the solutions regarding a given system of linear equations if provided with some information about the inverse of its coefficient matrix; and (c) the learners are able to find the inverse of a matrix using elementary row operations. The first two learning goals require the learners to possess sufficient conceptual knowledge regarding the relationships between inverse matrices, other types of matrices (e.g., identity matrices), and the matrix representation of a system of equations. The third goal requires the learners to memorize, recognize, and apply procedural knowledge about elementary row operations to find the inverse of a matrix.

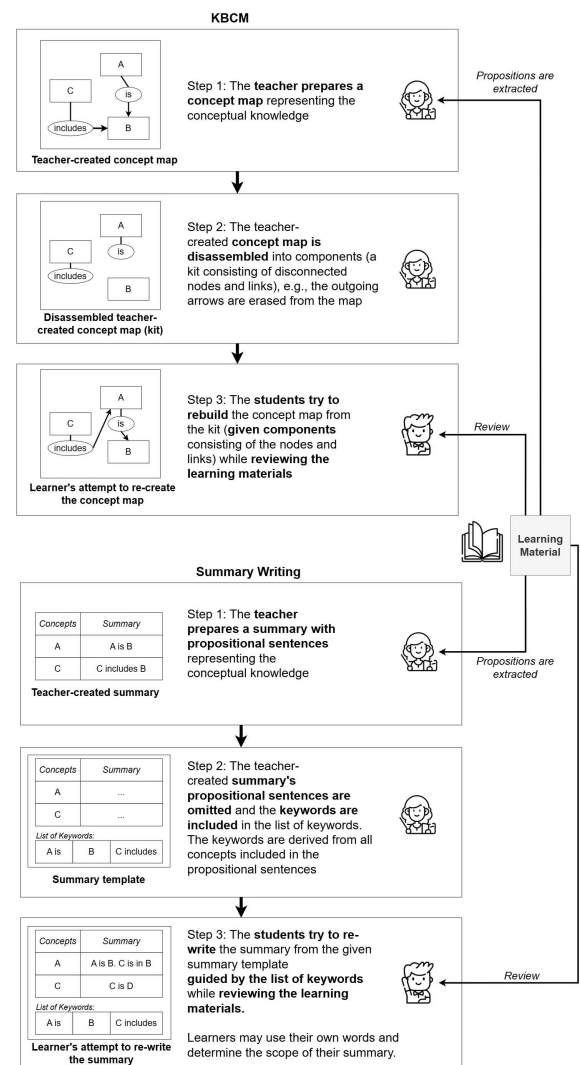


Fig. 1. Comparison of KBCM and summary writing activity. (Note: This figure was designed using images made by Freepik from Flaticon, <https://flaticon.com>).

Both the concept map and the summary template consisted of identical learning contents that included identical keywords and terms. However, KBCM and summary writing provided the learners with different information-organizing activities. In KBCM, a concept map is utilized; thus, a proposition (e.g., “If square matrix A does not have an inverse A^{-1} , the reduced row echelon of A has a zero row”) is visually represented with nodes (“square matrix A ” and “zero row”) and a link (“if does not have an inverse A^{-1} , the reduced row

echelon of it has”). In contrast, such a proposition is written as a full-sentence text in summary writing. A detailed description of how both activities differ is provided in Fig. 1.

As shown in Fig. 1, the teacher-created concept map is created to represent the conceptual knowledge included in the learning material. Fig. 2 shows the screenshot of the teacher-created concept map used in this study. Note that the concept map contains the propositions related to the mentioned learning topics.

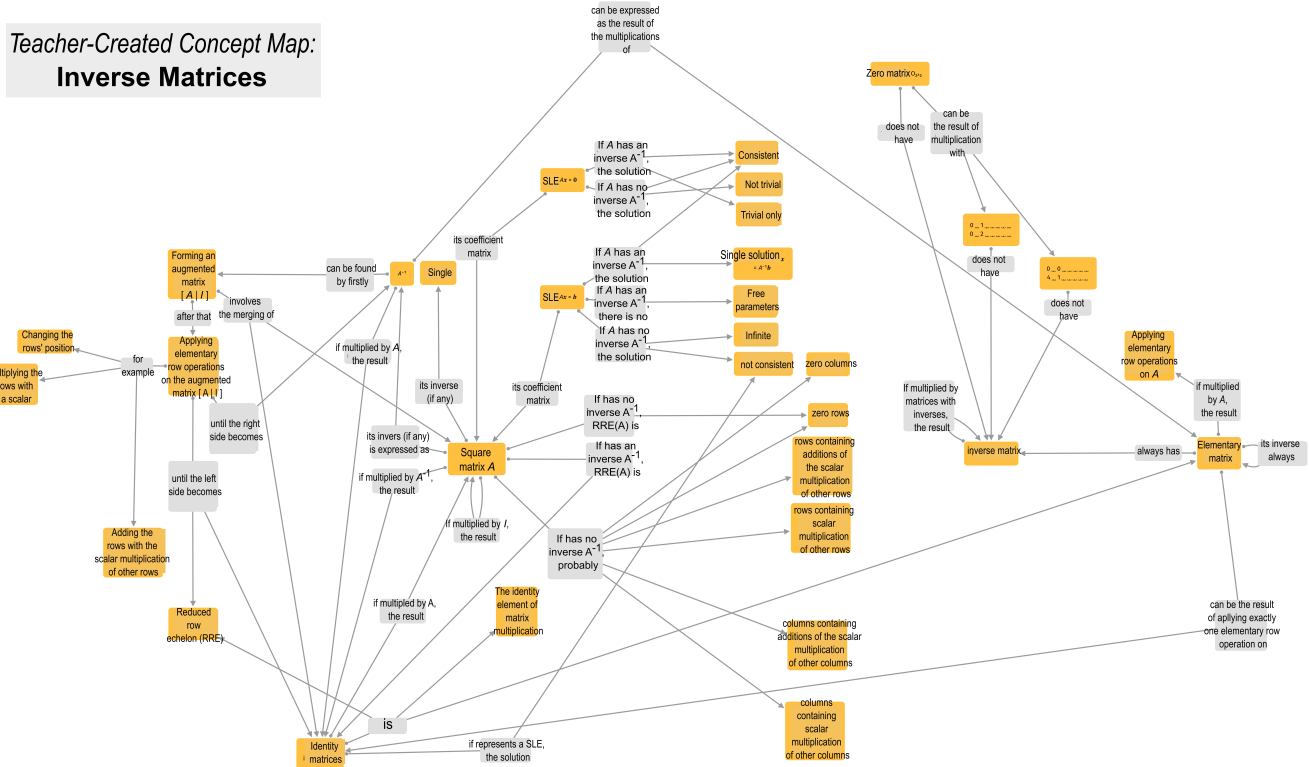


Fig. 2. The teacher-created concept map (translated into English from the original Bahasa Indonesia).

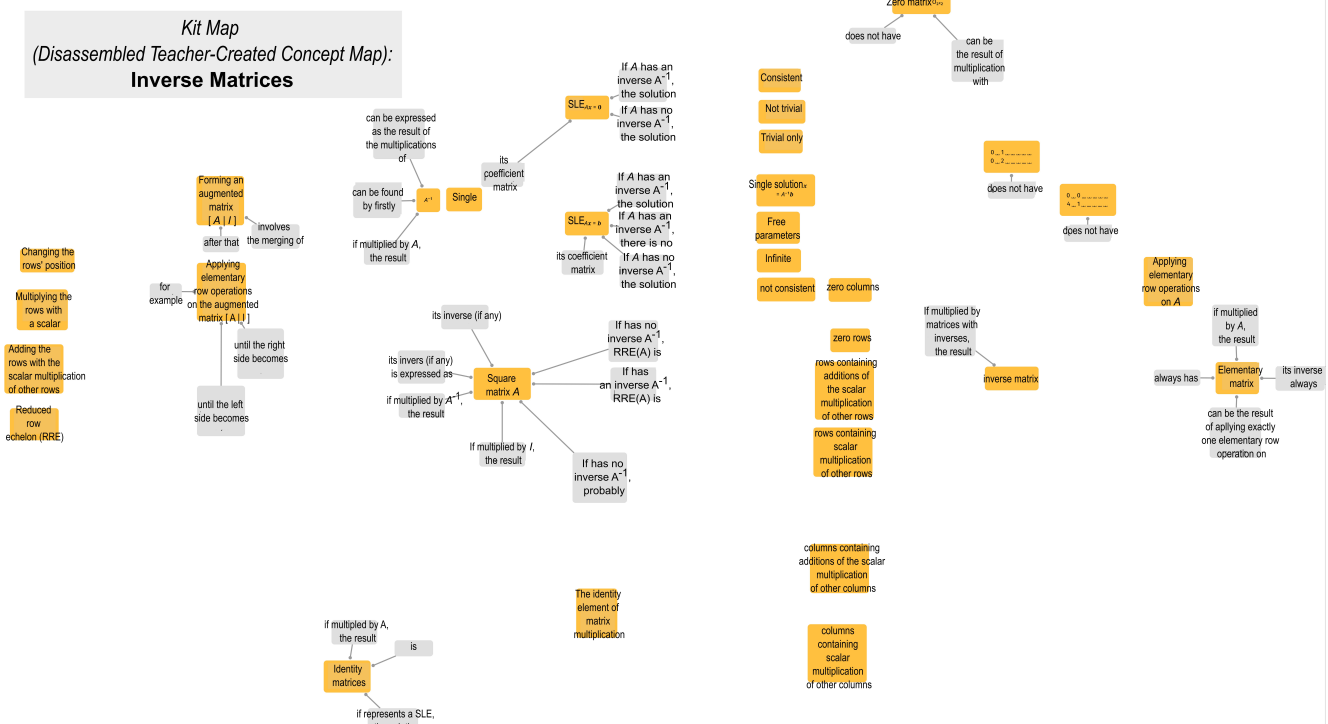


Fig. 3. The disassembled teacher-created concept map (translated into English from the original Bahasa Indonesia).

The disassembled teacher-created concept map (the kit) contains all the concepts in the teacher-created map. The

students are instructed to try rebuilding the teacher-created concept map from the given disassembled map while reviewing the relevant learning materials. Fig. 3 shows the screenshot of the disassembled (kit) concept map used in this study. Note that the links are disconnected by erasing the outgoing arrows that form propositions. The students could move the nodes freely but could not modify the texts or delete the nodes.

Two sets of questions consisting of 10 multiple-choice questions were developed by the instructors based on the learning goals to measure learners' conceptual knowledge (5 questions) and procedural knowledge (5 questions) in the mentioned topic. There is only one correct answer in each question. Table 1 provides an excerpt of the questions and the related learning goal.

Table 1. Excerpt of the questions included in pre-test and post-test

Type of Knowledge	Learning Goal	Question
Conceptual	(a) the learners can describe the characteristics of inverse matrices	Given A and B are square matrices; $AB = O$, thus, we can infer that ... a. Either A or B is a zero matrix b. Either A or B does not have an inverse c. A and B have no inverse d. Either A or B is an elementary matrix
	(b) the learners are able to infer the nature of the solutions regarding a given system of linear equations if provided with some information about the inverse of its coefficient matrix	Which of the following statement is equivalent with the statement: " $A_{n \times n}$ has an inverse"? a. $Ax = b$ has infinite solutions b. $Ax = b$ has a coefficient matrix that is row-equivalent with $I_{n \times n}$ c. $Ax = 0$ has a trivial solution d. $Ax = b$ has no solution
Procedural	(c) the learners are able to find the inverse of a matrix using elementary row operations	Given the following matrix: $B = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ Which sequence of elementary row operation is the correct one for obtaining B^{-1} ?
		a. $R_1 \leftarrow R_1 - R_2$ $R_2 \leftarrow R_2 - R_1$ $R_1 \leftarrow R_1 - 2R_2$ $B^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$ b. $R_1 \leftarrow R_1 - R_2$ $R_2 \leftarrow R_2 - R_1$ $R_1 \leftarrow R_1 - 2R_2$ $B^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$ c. $R_2 \leftarrow R_2 - R_1$ $R_1 \leftarrow R_1 - R_2$ $R_1 \leftarrow R_1 - 2R_2$ $B^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$ d. $R_1 \leftarrow R_1 - R_2$ $R_2 \leftarrow R_2 - R_1$ $R_1 \leftarrow R_1 - 2R_2$ $B^{-1} = \begin{bmatrix} 3 & -5 \\ 1 & -2 \end{bmatrix}$

As shown in Table 1, the questions for measuring conceptual knowledge were designed such that learners needed to recognize the relationships between some linear algebra concepts and infer the correct conclusion. On the other hand, the procedural knowledge questions required the learners to apply an arithmetical procedure correctly. The mentioned set of questions was administered to the learners during the pre-test and the post-test.

To ensure content validity, two linear algebra instructors created and reviewed the questions based on the two types of knowledge and learning goals (see Table 1). Both instructors had more than five years of experience teaching linear algebra to computer science students.

In addition, to ensure the questions' reliability, we also conducted a test-retest reliability procedure by calculating the correlation (Spearman ρ) between the pre-test and post-test scores of both the conceptual and procedural knowledge questions. Test-retest reliability measures how consistent the scores are if the test items are given at different times to the same group of people [61]. The set of questions used for measuring the mentioned type of knowledge (conceptual or procedural) also measures learners' understanding of various subtopics in linear algebra (e.g., measuring multiple learning goals as shown in Table 1). Thus, Cronbach's alpha is not fit for use in this case due to its purpose to determine whether the items measure only one single construct [62]. Calculating the correlation coefficient is one of the methods used to establish test-retest reliability, and it has been used in some studies about instrument development, e.g., Refs. [63, 64]. However, it should be noted that the existence of treatment

between the pre-test and post-test might affect the stability of the scores. However, the reliability of the test items could be shown by a significant correlation between the pre-test and post-test scores, which show that the scores for the test items are stable to a certain degree.

The result of the test-retest reliability procedure showed that there was a significant and positive correlation between the pre-test and post-test scores of the conceptual knowledge questions ($p = 0.000$; $\rho = 0.430$). Additionally, there was a significant but weak positive correlation between the pre-test and post-test scores of the procedural knowledge questions ($p = 0.018$; $\rho = 0.230$). Despite showing various correlation magnitudes, the correlation coefficients for both types of questions are significant, which suggests that the treatment also contributed to the results of the post-test.

Regarding the sample sizes ($n_{KBCM} = 54$; $n_{summary} = 51$), we acknowledged that our sample was limited by resource constraints. The number of students in each class (the treated group) depended on the number of enrolled students, which was outside of the researchers' control. We conducted power analysis on the statistical tests to justify the sample size. Based on the results of the power analysis, some tests have >0.90 power, but some others have a small power (<0.50). Thus, this study has a limited sample size that makes future replication studies with a larger sample size and in different educational contexts necessary to confirm the findings. However, in this study, an interpretation regarding the potentials of the treatments (KBCM & summary writing) can be made by comparing the differences in effect size values.

B. Procedures and Data Analysis

The following procedures were conducted in this study: (a) First, the learners attended a lecture introducing both conceptual and procedural knowledge on the topic; (b) after the lecture, the pre-test measuring both types of knowledge was administered; (c) after completing the pre-test, the learners were instructed to organize their conceptual knowledge by either KBCM or summary writing based on their assignment; and (d) finally, the post-test was administered. The mentioned procedures are illustrated in Fig. 4.

During the two lecture sessions (1st lecture: 90 min; 2nd lecture: 50 min), both KBCM and summary writing groups were introduced to the topic of inverse matrices using the same learning materials and the same exercises. The instructors worked closely together to ensure the same treatment in the class at this point. Both groups also received training on the effective use of KBCM or summary writing to ensure that both groups were well-prepared to conduct the mentioned activities.

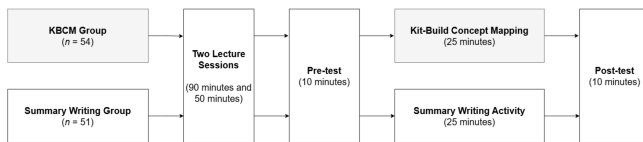


Fig. 4. Procedures.

After the pre-test was administered, the groups were given 25 min to complete the treatments. The KBCM group was given components of a disassembled concept map, while the summary writing group was given a structured summary writing template consisting of keywords for guiding the writing. Fig. 1 shows the detailed processes during KBCM and summary writing activities.

The pre-test and post-test scores were collected and analyzed using non-parametric statistical significance tests to answer the RQ (to what extent did conceptual and procedural knowledge improve?). The effect sizes (Pearson r) of the pre-test and post-test scores comparison were calculated to determine the extent of the impact on learning performance. Table 2 describes the interpretation of effect sizes based on Funder and Ozer [65].

Effect size r	Interpretation (based on Funder & Ozer [65])
0.05–0.099	Very small
0.100–0.199	Small
0.200–0.299	Medium
0.300–0.399	Large
≥ 0.400	Very large

Holm's correction method is also used in some of the comparisons between pre-test and post-test scores. The purpose is to minimize the likelihood of getting significant p -values by chance (Type I error) due to conducting multiple comparisons (e.g., comparing the pre-test and post-test scores between the KBCM and the summary writing groups as well as comparing the improvement within each group). The adjusted p -values are presented in some of the comparisons in the Results section.

In addition, a correlation analysis was conducted to obtain the Spearman ρ coefficients that show the degree of

correlation between the KBCM group learners' map score and the post-test scores of (a) conceptual knowledge and (b) procedural knowledge. The map score shows the correctness of the attempt to reconstruct the teacher-created map, which was calculated by counting the number of correct propositions divided by the total number of propositions existing in the teacher-created concept map. The map scores were standardized into 0–100 points. The correlation analysis was conducted to check to what extent the relationship between the mentioned reconstruction correctness and learning achievement differs. The interpretation of the correlation magnitude was made based on the work of Dancey and Reidy [66]. Table 3 shows the category for interpreting the magnitude of the correlation.

The purpose of conducting the correlation analysis was to check the assumption that attaining well-organized conceptual knowledge is related to better performance in solving problems that require conceptual understanding.

Table 3. Correlation magnitude interpretation

Spearman ρ coefficients	Interpretation (based on Dancey & Reidy [66])
$\geq 0.70.00$	Very strong
0.40–0.69	Strong
0.30–0.39	Moderate
0.20–0.29	Weak
0.01–0.19	Negligible or none

The interpretation applies for both positive and negative coefficients.

IV. RESULTS

The descriptive statistics of the pre-test scores, post-test scores, and standardized learning gain for conceptual and procedural knowledge are shown in Table 4. Some data are not normally distributed.

Table 4. Descriptive statistics of pre-test scores, post-test scores, and learning gain

Group	Type of Knowledge	Data	Mean	Median	Min	Max
KBCM ($n = 54$)	Conceptual	Pre	49.07	50.00	0.00	100.00
		Post	63.27	66.67	16.67	100.00
		Gain	0.25	0.25	–0.67	1.00
	Procedural	Pre	35.65	25.00	0.00	100.00
		Post	58.80	62.50	0.00	100.00
		Gain	0.31	0.33	–1.00	1.00
SW ($n = 51$)	Conceptual	Pre	56.86	66.67	16.67	83.33
		Post	56.54	66.67	16.67	83.33
		Gain	0.01	0.00	–1.00	0.67
	Procedural	Pre	30.88	25.00	0.00	100.00
		Post	56.37	50.00	0.00	100.00
		Gain	0.32	0.33	–1.00	1.00

KBCM: Kit-Build Concept Mapping; SW: Summary writing.

For comparing the extent of the learners' understanding of conceptual knowledge, the pre-test scores as well as the post-test scores of both KBCM and summary writing groups were analyzed using non-parametric statistical significance tests (Mann-Whitney test & Wilcoxon paired-samples test). The p -values observed from each comparison of pre-test and post-test scores were adjusted using Holm's method. Fig. 5 shows the box plot diagrams that include the results of the significance test.

As shown in Fig. 5, the results suggest that both the KBCM and summary writing groups started at the same level of conceptual knowledge understanding as there was a non-significant difference in the pre-test scores after Holm's method adjustment ($p = 0.029$; $p_{adj} = 0.087$; effect size

$r = 0.210$ [medium]; power = 0.187). This finding shows that both groups had the same level of conceptual knowledge after attending the lecture sessions, despite the fact that both groups were taught by different teachers. This suggests that the teacher differences may not have had a substantial influence on the scores.

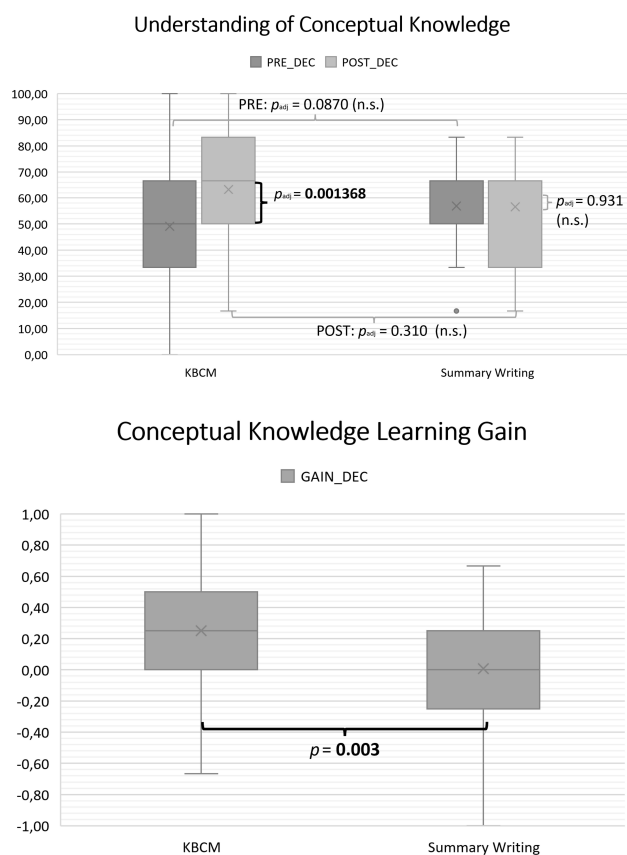


Fig. 5. Conceptual knowledge pre-post test scores and learning gain comparisons.

After the treatment, both groups also showed no significant differences in the level of conceptual knowledge ($p = 0.155$; $p_{adj} = 0.310$; effect size $r = 0.140$ [small]; power = 0.109). However, despite the seemingly same level of understanding after the activity, both groups showed significantly different learning gains, with a higher gain observed among learners who did KBCM ($p = 0.003$; effect size $r = 0.290$ [medium]; power = 0.313). In addition, only the KBCM group experienced a significant increase in the test score ($p_{adj} = 0.001,368$; effect size $r = 0.525$ [very large]; power = 0.966). These results indicate that KBCM has the potential to contribute to the improvement in conceptual knowledge understanding with a significant but moderately higher learning gain.

As for procedural knowledge, both groups also showed no significant differences in the pre-test and post-test scores ($p = 0.344$; $p_{adj} = 0.688$; effect size $r = 0.090$ [very small]; power = 0.074). Both groups showed significant improvement in the test scores (KBCM: $p = 0.000,004,18$; $p_{adj} = 0.000,016,72$; effect size $r = 0.633$ [very large]; power = 0.995; Summary writing: $p = 0.000,007,53$; $p_{adj} = 0.000,022,59$; effect size $r = 0.644$ [very large]; power = 0.996) but without a significant difference in their standardized learning gains ($p = 0.974$; effect size $r = 0.000$; power = 0.050). These results indicate that both KBCM and

summary writing contributed to a significant and very large improvement in procedural knowledge understanding. Fig. 6 shows the results of the procedural knowledge.

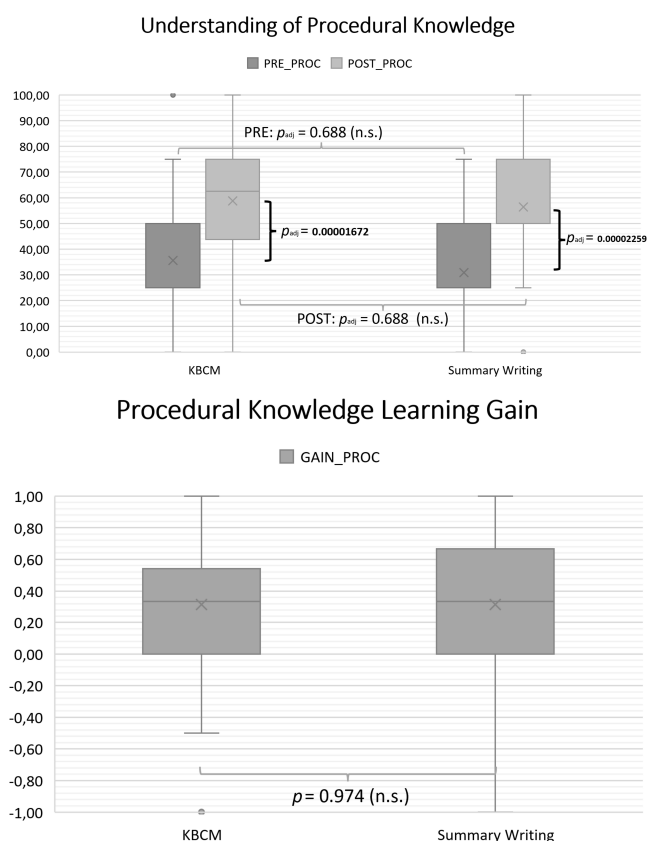


Fig. 6. Procedural knowledge pre-post test scores and learning gain comparisons.

The results shown in Fig. 6 show that most of the learners in the KBCM group experienced an increase in their test scores for conceptual knowledge, whereas, in the summary writing group, there were learners who experienced such an increase as well as a decrease in test scores. This suggests that the learners in the summary writing group had difficulties answering the questions that required them to effectively recognize relationships between several concepts within the domain of linear algebra. In addition, during the post-test, they might have relied on retrieving their previous answer from memory, thus retaining their prior understanding, which might include some misconceptions. In contrast, most of the learners in the KBCM group might have been helped by the activity of reconstructing the concept map, which enabled them to recognize the relationships between various mathematical concepts, memorize them, and retrieve them when needed to answer the post-test questions. In addition, as opposed to the results of the conceptual knowledge, most of the learners in both groups could potentially be able to effectively conduct the necessary mathematical procedures (see Fig. 6).

There was a possibility that the KBCM group learners' attempt at reconstructing the teacher-created map correlated with their performance at solving conceptual knowledge-related problems. To check this possibility, the correlation coefficients (Spearman ρ) for the relationship between the map score of the learners from the KBCM group and their post-test scores for both types of knowledge were obtained. Table 5 shows the result of the correlation analysis.

Table 5. Correlations between conceptual knowledge post-test scores, procedural knowledge post-test scores, and map score

Variables	Map Score	CPOST	PPOST
Map score	-	0.54 ($p = 0.0000$) ***	0.39 ($p = 0.0037$) **
CPOST	($p_{adj} = 0.0000$) ***	-	0.24 ($p = 0.0790$) n.s.
PPOST	($p_{adj} = 0.0055$) **	($p_{adj} = 0.0790$) n.s.	-

CPOST: Conceptual Knowledge Post-Test Scores; PPOST: Procedural Knowledge Post-Test Scores; The Spearman ρ coefficients are displayed outside the brackets in the upper diagonal; The original p values are shown inside brackets in the upper diagonal; The adjusted p values (after BH method adjustment) are shown in the lower diagonal;

* $p_{adj} < 0.05$; ** $p_{adj} < 0.01$; *** $p_{adj} < 0.001$; n.s.: not significant.

The results shown in Table 5 indicate that there are positive and significant correlations between KBCM learners' map scores and the post-test scores of both conceptual and procedural knowledge. The result from the correlation analysis indicates that the KBCM group learners' attempt at reconstructing the teacher-created map correlated with their performance at solving problems that require the understanding of conceptual knowledge or procedural knowledge, but the relationship between the map reconstruction attempt and their performance at answering conceptual knowledge-type questions was stronger. The correlation between the map score and conceptual post-test score ($\rho = 0.54$; strong) was found to be higher compared to the correlation between the map score and procedural post-test score ($\rho = 0.39$; moderate). Note that this interpretation is based on the work of Dancey and Reidy [66].

V. DISCUSSION

To answer the research question (to what extent did conceptual and procedural knowledge improve?), the results show that conceptual knowledge was significantly improved among learners who did KBCM and that the provision of KBCM significantly contributed to such improvement. As shown in Fig. 5, after the treatment, only the KBCM group showed significant improvement in test scores with a very large effect size ($p_{adj} = 0.001368$; effect size $r = 0.525$). In addition, there is no significant difference in the pre-test scores of KBCM and summary writing groups for both conceptual and procedural knowledge tests, which indicates that both groups had the same level of prior knowledge for both conceptual and procedural knowledge before the treatments, and learners' prior knowledge was not the sole factor affecting the learning outcomes (i.e., the treatments also contributed to the learning outcomes).

The results presented in Fig. 5 also showed that the learners in the KBCM group also exhibited a significantly higher rate of improvement in their conceptual understanding, which is shown by a significantly higher median of standardized learning gain with medium effect size ($p = 0.003$; effect size $r = 0.290$). In comparison, those who did summary writing did not show significant improvement in conceptual knowledge, in addition to the finding that some learners even showed a decrease in their test scores due to potentially retained misconceptions. Therefore, KBCM has the potential to effectively improve conceptual knowledge in mathematics.

In addition, procedural knowledge was also significantly improved in both groups, which may be the result of the learners' priorly acquired procedural knowledge as shown by the pre-test scores (e.g., sufficient memorization of

procedures). As shown in Fig. 6, both the KBCM and summary writing groups experienced significant improvements in test scores with very large effect sizes ($p_{adj} > 0.001$; effect size $r > 0.600$).

A. Characteristics of KBCM and Summary Writing Activity that Might Affect the Results

Two characteristics of KBCM may explain the improvement of conceptual knowledge: (a) the provision of visually displayed components (nodes & links); and (b) the activity of connecting the provided components to form propositions. The visual display of knowledge structure is the general feature of a concept map, which has been suggested as beneficial for learning in many educational contexts [67–69]. Concept mapping may enable learners to progress from mere memorization or rote learning toward deeper conceptual learning [70]. Moreover, the nature of the concept map as an externalized representation of knowledge has been recognized as an effective tool for assessing understanding of conceptual knowledge [71, 72]. Therefore, past studies suggested the relationship between making concept maps and the improvement of conceptual understanding.

In the context of KBCM, at the end of the activity, the learners might catch the overall structure (e.g., the scope of contents and the relationships between concepts) easily by viewing the visual representation of the knowledge structure. In line with Ref. [73], it might be easier to catch the relationships between concepts by recognizing the interconnected nodes in a concept map compared to catching the implicit relationships within a bunch of text passages. Each node represents an idea (e.g., a mathematical object) that allows it to be connected with other ideas to form a web of understanding in the form of concept-relationship-concept [2, 74]. Therefore, being able to catch such a relationship may enable learners to effectively gain conceptual knowledge by extracting meanings from the learning materials (e.g., how a mathematical object could be related to other relevant concepts under certain conditions).

In contrast, as for writing propositional sentences in a summary, many important concepts might be written in different places with different terms [74]—note that a learner might or might not use the list of keywords provided when trying to rewrite the teacher-created summary and end up writing some wrong propositions (see Fig. 1). Therefore, it might be harder for the learners who wrote summaries to catch the relationships between the concepts correctly, which led to difficulties answering the conceptual knowledge questions.

Extracting and interpreting information from learning material—as typically conducted in from-scratch concept mapping and summary writing are tedious tasks that demand a high cognitive load due to requiring learners to understand encountered terms (decode meaning), categorize them, recognize relationships, and memorize them. Thus, a different amount of cognitive load exerted from making a summary and conducting KBCM might have influenced the learning outcomes despite the fact that the learners were provided with identical keywords, which were related to the learning content, in both activities as hints for making propositions, e.g., as components (nodes & links) in KBCM

and as a list of keywords in the summary writing template.

This difference might have been caused by the different characteristics of the two mentioned activities. For example, in KBCM, the provision of nodes and links as scaffolds might lessen the extrinsic cognitive load because learners do not need to extract the knowledge structure from the learning materials by themselves when attempting to organize information. This was suggested in a related prior study comparing the effectiveness of KBCM and summary writing for improving overall learning outcomes in mathematics [28].

In many situations, learners who are instructed to write a summary about learning material often have difficulties expressing the material in their own words and organizing it; thus, it may lead to an overwhelming cognitive load and even the detrimental behavior of copy-and-paste [75, 76]. Such an activity might deter learners from effectively catching the relationship between concepts within a knowledge domain. Particularly, giving summary writing assignments to lower-achieving students may lead to potential difficulty in understanding various terms used in the learning materials and using them in paraphrases.

The findings of this study, of course, do not necessarily suggest that summary writing is ineffective for improving conceptual understanding in many situations. Summary writing has long been considered a beneficial learning activity [17]. Some of its educational potential includes improved content comprehension [15, 16]. However, despite its potential, providing a summary writing activity does not automatically lead to a high-quality summary showing a deep understanding of the content. In regards to this, there is a possibility of different learners' ability to: (a) actively decode meaning from a text; (b) connect them with what they already know as well as realize the connection between sentences (or propositions); (c) and paraphrase them while at the same time trying to check their own understanding. Such processes are considered effortful, and achieving a deep level of understanding from those activities is not adequately addressed by many instructors [16]. Thus, summary writing effectiveness might depend on the learners, i.e., some can write concisely and correctly in a well-organized way, but some cannot.

In contrast to writing propositions from scratch in summary writing, in the case of KBCM, actively connecting the given concept map components to form the correct propositions might have helped the learners in the KBCM group to catch the structure of the knowledge domain without requiring them to extract the propositions from the learning materials into another form of knowledge representation by themselves, e.g., there is no need to paraphrase the sentences from the learning materials. As a result, the following processes might help learners of KBCM to catch the structure of the knowledge: (a) decoding meaning triggered by the given components (e.g., only the correct terms are provided in the links and nodes); and (b) realizing the correct connection between propositions more easily due to actively trying to connect the links in the concept map with the support of immediate feedback. In other words, KBCM learners might be more aware of the structure of the knowledge domain, could effectively recognize the interconnected concepts within the learning topic, and achieve a better-organized understanding compared to those in the

summary writing group. This is in line with Novak and Cañas [20], who suggested that meaningful learning could be achieved through attaining a well-organized understanding of the knowledge structure. Therefore, KBCM learners could potentially be able to answer questions that require them to relate one concept to another and catch the correct meaning behind such a relationship.

Moreover, the existence of a significantly positive correlation between the map score and the post-test score of conceptual knowledge (see Table 4) supports the assertion regarding the existence of a relationship between the concept mapping attempt and conceptual knowledge acquisition. As for the activity of creating concept maps, a prior study suggested that if learners are able to make high-quality concept maps, they may show better learning performance due to acquiring a well-organized and deep understanding from making accurate connections between concepts [35, 77]. In regards to this assertion, KBCM activity involves the use of a teacher-created map, which can be a well-defined externalized representation of the knowledge domain with correct propositions and terms. By actively reconstructing the teacher-created concept map from given components, learners might be enabled to recreate a high-quality concept map, thus maximizing their potential to acquire a complex understanding of the topic represented in the map. This is in line with a past study that discussed the relationship between the ability to make a complex concept map and learning achievement [78].

Despite the finding of significantly better learning performance in the KBCM group in the case of solving conceptual knowledge-related questions (see Fig. 5), further investigation on whether KBCM learners may or may not outperform those who use traditional scratch concept mapping is needed. The traditional scratch concept mapping may be more difficult for some learners if no support is provided, e.g., multimedia as a mental scaffold, reading sequence, etc. [74, 79]. Both scratch concept mapping and the summary writing may require learners to extract the knowledge structure by themselves; thus, there is a possibility that some learners may experience difficulty in effectively organizing their understanding.

B. Practical Implications

Understanding the relationships between concepts might have contributed to the significant increase in the conceptual knowledge test score in most KBCM learners as well as the significantly higher conceptual knowledge learning gain (see Fig. 3). Based on the results, the following recommendations may be considered for improving the teaching of a mathematics subject, especially in a higher-educational context that requires deeper conceptual learning.

- 1) Firstly, providing KBCM that includes a well-designed teacher-created concept map as an after-lecture activity for organizing understanding has the potential to benefit learners by enabling them to correctly recognize the concepts and their relationship (knowledge structure) within a topic; thus, it may help them answer conceptual knowledge-related questions;
- 2) Secondly, providing other forms of knowledge representations or activities may be needed for a better improvement of procedural knowledge.

VI. CONCLUSION

Both conceptual and procedural knowledge are important in learning mathematics. However, improving conceptual knowledge is a challenging task due to the many reports of learners being able to do mathematical procedures without knowing the reasons or the principles that make them work. Providing an information-organizing activity may help learners organize their understanding after being introduced to conceptual knowledge in a lecture session. However, it is challenging to provide the traditional way of writing a summary with a structured template for organizing information to catch the relationships between concepts included in a topic due to the possibility of a high cognitive load.

This study proposed the use of a KBCM as an alternative after-lecture activity to help learners easily recognize the structure of the knowledge domain and increase their conceptual understanding of a topic in a mathematics subject. KBCM is expected to help learners understand the structure of the knowledge (e.g., becoming aware of the interconnected concepts within a domain) without requiring them to extract and express the structure by themselves, as in writing a summary; thus, it is expected to have the potential to effectively help learners answer conceptual questions that require them to be aware of the connections between concepts. An experiment was conducted involving 105 students in two linear algebra classes at a public university. Both conceptual and procedural knowledge were tested before and after the intervention to compare the effectiveness of the KBCM and traditional summary writing for the mentioned purpose.

To answer the RQ (to what extent did conceptual and procedural knowledge improve?), in this case, the learners who did KBCM experienced a significant improvement in conceptual knowledge compared to the control group, who did summary writing, which is shown by significantly higher learning gain. KBCM that includes conceptual knowledge has the potential to effectively improve mathematical conceptual and procedural knowledge. However, a better intervention for improving mathematical procedural knowledge still needs further investigation. However, the use of KBCM and summary writing might not be the sole factor that affected the scores due to the fact that the experiment was conducted in a real classroom setting. Therefore, the results could not yet be generalized. This study is limited to a case study involving the implementation of KBCM and summary writing for teaching a mathematics subject. A future study needs to take into account several other potentially influencing factors that could affect the effectiveness of the mentioned interventions.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

AUTHOR CONTRIBUTIONS

LMH: conceptualization, methodology, formal analysis, investigation, data curation, visualization, writing—original draft; KJ: conceptualization, validation, resources, project administration; LS: conceptualization, validation, resources, project administration; TH: conceptualization, supervision;

YH: conceptualization, methodology, investigation, writing—review and editing, supervision, project administration, funding acquisition; all authors had approved the final version.

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